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A Method for Reduction of Number of Actuators in Independent Modal Space Control

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In this paper, a modified Independent Modal Space Control (IMSC) that relaxes the fundamental hardware requirement of IMSC is proposed for handling the vibration and attitude control problem of large, flexible structures. The method incorporates a new switching algorithm for dynamically selecting controlled modes and a novel design technique for determining the modal control force. The main advantage of the proposed method is that it minimizes the discontinuity of the modal control forces and assures the asymptotic stability of the closed-loop system. The simplicity and efficiency of the method is demonstrated through an example involving vibration control of a cantilevered beam. The system performance and stability of the proposed method is compared with previously published methods that also seek to reduce the number of actuators in IMSC.

Key Words: Vibration Control, Asymptotic Stability, Switching Algorithm, Number of Actuators, IMSC

1. Introduction

Many control methods have been proposed to date for controlling the vibration of large, flexible systems such as space structures (Balas, 1978; 1979; Chen, 1982; Goh and Coughey, 1985; Fanson and Caughey, 1990). One widely known method is collocated control (Balas, 1978; 1979; Chen, 1982) that uses direct velocity feedback. This method is unconditionally stable if the actuator dynamics can be neglected, but can be unstable in the presence of actuator dynamics. To remedy this deficiency, Goh and Caughey (Goh and Caughey, 1985; Fanson and Caughey, 1990) have proposed positive position feedback control. This method maintains stability even in the presence of significant actuator dynamics. In addition, the stability criterion is not affected by variations in the system parameters.

Although numerous other methods have also been reported in the open literature, they all face implementation difficulties as the order of the discretized model of the system increases. For instance, for cases where the control gains need to be calculated in real-time, the amount of computation required may become prohibitive for high-order control systems. To address this problem, Meirovitch and coworkers (Meirovltch and Baruh, 1982; 1983; Meirovitch, 1990; Meirovitch et al., 1983; Hale and Rahn, 1984; Baruch and Silverberg, 1985; Baz et al., 1992) have proposed the Independent Modal Space Control (IMSC) method that can design controllers in mutually independent modal space with relative ease and simplicity. In this method, the modal matrix serves as the transformation matrix that transforms the equations of motion of a structure into a set of decoupled equations in modal coordinates. The controller is then individually designed for each modal coordinate. Since the controller design is performed for each mode based on the

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respective modal states, the design process is quite simple, irrespective of the degree of freedom of the system. In particular, the amount of computation is drastically reduced. The modal control force thus obtained can then be converted to the actual control force through coordinate transformation.

For IMSC, the controllability is always assured, and the control spillover can be minimized, provided that the number of actuators is equal to the number of modes. In estimating modal states for determining the modal control force, the observation spillover problem can be resolved by employing a modal filter with sufficient number of sensors (Hwang et al., 1998). Notwithstanding these advantages, the requirement that the number of actuators be equal to the number of controlled modes places a fundamental hardware constraint that tends to limit the applicability of the method. The main focus of the present work is to relax the hardware requirement by reducing the number of actuators within the context of IMSC (Meirovitch, 1990; Baz et al., 1992; Lindberg and Longman, 1984; Baz and Poh, 1988). The situations involving fewer number of actuators can arise either by designing in fewer actuators or through loss of actuators due to some damage sustained during operation. To achieve vibration control with fewer actuators than the number of modes, either a method of control force computation based on pseudo-inverse or a method involving selective modal control based on some switching algorithm can be employed.

In this paper, a switching algorithm that can reduce the number of actuators without appreciable deterioration in control system performance is developed. A novel controller design incorporating this algorithm is then proposed and validated. When fewer actuators are employed, switching may cause instability due to energy transfer between the system modes. Therefore, the stability is also examined and verified with the switching algorithm incorporated into the control scheme.

2. Reduction of the Number of Controllers in the Independent Modal Space Control

The discretized equations of motion of continuous systems can be approximated by an ordinary differential equation of the form given by

$$I\ddot{\nu}(t)+\Lambda\nu(t)=f(t) \qquad (1)$$

where I is an identity matrix of order n, Λ a diagonal matrix whose diagonal elements are squares of the natural frequencies ($\Lambda = diag[\omega_1^2, \cdots, \omega_n^2]$), and $f(t)$ denotes the modal control force vector of order n. The order n refers to the number of system modes to be controlled, and $\nu(t)$ is the modal displacement vector needed for controller design. In deriving Eq. (1), the effect of structural damping is neglected. Since the control force for each mode is independently designed when using IMSC, the design process is quite simple. However, the number of actuators should be equal to the number of system modes (Meirovitch and Boruh, 1982; 1983; Meirovitch, 1990), placing a fundamental hardware constraint on the controller design. Although some efforts at reducing the number of actuators have been reported (Meirovitch, 1990; Baz et al., 1992; Lindberg and Longman, 1984; Baz and Poh, 1988), more work is needed in this regard. For example, Meirovitch et al. (1990) showed that the number of actuators could be reduced by using pseudo-inverses. This method has proven to be ineffective with regard to system performance and stability, however. Lindberg and Longmen (1984) have designed controllers in IMSC based on a suboptimal control method. However, the resulting solution is not unique, and the stability of the closed-loop system cannot be guaranteed. Thus, verification is needed for each solution. Baz and Poh (1988) have proposed a modified IMSC based on dynamic switching of actuators between the system modes. At each time step, the magnitudes of the modal energy are compared, and the modes equal to the number of actuators are selected for control. The selection is based on the instant energy level possessed by each mode; the modes with high energy levels are selected. This method entails difficulties associated with the switching algorithm, and stability cannot be guaranteed due to asymmetry of the coefficient matrices of the closed-loop system equations. To address these problems, an unconditionally stable IMSC method utilizing fewer actuators than the number of the system modes and applying a new switching algorithm is presented. A brief summary and evaluation of the pseudo-inverse method and the method of Baz and Poh is presented first to place the proposed method in a proper context.

2.1 Pseudo-Inverse Method

As the name implies, IMSC entails obtaining the modal control force $f(t)$ in modal space. To obtain $f(t)$, various methods such as optimal control can be applied independently for individual modes. By the modal expansion theorem, the following relationship between the modal control force vector and the actual control force vector can be obtained:

$$f(t)=BF(t), \quad (2)$$
$$B=[\phi_i(x_j)] \quad i=1,2,\cdots,n, \ j=1,2,\cdots,m,$$

where $F(t)$ denotes the actual control force vector, $\phi_i(x_j)$ the value of i-th eigenfunction at x_j, n the number of system modes, and m the number of actuators. In order to express $F(t)$ as a function of $f(t)$, matrix B must be inverted. Since the inverse can only be obtained if the number of actuators and system modes are equal to each other, i. e., $m=n$, use of IMSC requires that the same number of actuators be used, severely limiting the usefulness of the method for practical applications involving large number of modes.

With fewer actuators ($m<n$), the following pseudo-inverse can be used to obtain the actual control force (Meirovitch, 1990),

$$F(t)=(B^TB)^{-1}B^Tf(t)=B^+f(t) \quad (3)$$

where B^+ denotes the pseudo-inverse of B. Since the pseudo-inverse is not the real inverse, however, error is present. Therefore the modal control force vector $f(t)$ deviates from the nominal design value, and significant error in the system response is possible. Furthermore, the closed-loop system can become unstable.

2.2 Modified Independent Modal Space Control Method of Baz and Poh

In order to reduce the number of actuators, Baz and Poh have proposed a modified method, in which the allocation of the modes to be controlled by actuators are determined by the energy levels of the system modes. At each time step, instant modal energy levels are compared, and switching of modes for actuator application occurs based on the latest ordering (Baz and Poh, 1988). In other words, switching occurs at every fixed time interval, and the switching time is identical for all modes. In order to more closely examine the potential problems associated with this method, a slight rephrasing is performed as follows. First, the modal control force vector can be written as

$$f(t)=\Gamma f^*(t) \quad (4)$$

where the columns of the coordinate transformation matrix Γ are composed of the columns of the identity matrix and serves to reorder the modal control force vector, and $f^*(t)$ represents the reordered modal control force vector for the new time step. The modes to be controlled and neglected can be written as

$$f^*(t)=\begin{bmatrix} f_C(t) \\ f_U(t) \end{bmatrix} \quad (5)$$

where $f_C(t)$ denotes the vector composed of the controlled modes and $f_U(t)$ the vector of the modes neglected for the time being, i. e., until the next time step. In general, $f_U(t)$ represents the modal forces arising from the control spillover. The modal control force vector can be converted to the actual control force through the modal transformation matrix composed of the eigenfunctions

$$f^*(t)=\begin{bmatrix} \phi_1(x_1) & \cdots & \phi_1(x_m) & \phi_1(x_{m+1}) & \cdots & \phi_1(x_n) \\ \vdots & & \vdots & \vdots & & \vdots \\ \phi_m(x_1) & \cdots & \phi_m(x_m) & \phi_m(x_{m+1}) & \cdots & \phi_m(x_n) \\ \phi_{m+1}(x_1) & \cdots & \phi_{m+1}(x_m) & \phi_{m+1}(x_{m+1}) & \cdots & \phi_{m+1}(x_n) \\ \vdots & & \vdots & \vdots & & \vdots \\ \phi_n(x_1) & \cdots & \phi_n(x_m) & \phi_n(x_{m+1}) & \cdots & \phi_n(x_n) \end{bmatrix}.$$

$$\begin{bmatrix} F_C(t) \\ F_U(t) \end{bmatrix}=\begin{bmatrix} B_{CC} & B_{CU} \\ B_{UC} & B_{UU} \end{bmatrix}\begin{bmatrix} F_C(t) \\ F_U(t) \end{bmatrix} \quad (6)$$

where the subscripts C and U denote the controlled and uncontrolled modes, respectively. Since the number of actuators is equal to m, $F_U(t)=0$ and $f^*(t)$ can be written as

$$f^*(t)=\begin{bmatrix} B_{CC} \\ B_{UC} \end{bmatrix} F_C(t). \tag{7}$$

The actual control force vector is given by

$$F_C(t)=B_{CC}^{-1} f_C(t) \tag{8}$$

and the modal control force vector for the uncontrolled modes is given by

$$f_U(t)=B_{UC}B_{CC}^{-1} f_C(t) \tag{9}$$

The above development demonstrates that employing fewer actuators than the number of modes entails control spillover $f_U(t)$ for the uncontrolled modes.

The modal control vector in independent modal space can be represented by

$$f_C(t)=-K_p \nu_C(t)-K_V \dot{\nu}_C(t) \tag{10}$$

where K_p and K_V are the positive definite control gain matrices. Substituting Eqs. (8), (9) and (10) into Eq. (7), we obtain

$$f^*(t)=\begin{bmatrix} -K_p & -K_V \\ -B_{UC}B_{CC}^{-1}K_p & -B_{UC}B_{CC}^{-1}K_V \end{bmatrix}\begin{bmatrix} \nu_C(t) \\ \dot{\nu}_C(t) \end{bmatrix}$$

$$=\begin{bmatrix} K_p & 0 \\ B_{UC}B_{CC}^{-1}K_p & 0 \end{bmatrix}\nu^*(t)-\begin{bmatrix} K_V & 0 \\ B_{UC}B_{CC}^{-1}K_V & 0 \end{bmatrix}\dot{\nu}^*(t)$$

The above equation can be rewritten as

$$f^*(t)=-G_p\nu^*(t)-G_V\dot{\nu}^*(t) \tag{11}$$

where

$$G_p=\begin{bmatrix} K_p & 0 \\ B_{UC}B_{CC}^{-1}K_p & 0 \end{bmatrix},\ G_V\begin{bmatrix} K_V & 0 \\ B_{UC}B_{CC}^{-1}K_V & 0 \end{bmatrix}$$

and $\nu^*(t)$ denotes the reordered modal displacement vector in which the controlled and uncontrolled modes are put in the form of

$$\nu^*(t)=\begin{bmatrix} \nu_C(t) \\ \nu_U(t) \end{bmatrix}=\Gamma^T\nu(t). \tag{12}$$

There are two main problems associated with the method of Baz and Poh. First, the coefficient matrices of the closed-loop equations, i. e., $\Gamma G_p \Gamma^T$ and $\Gamma G_V \Gamma^T$, become asymmetric. It is known that systems possessing asymmetric coefficient matrices cannot be unconditionally stable. Second, the switching of actuators occurs after each time step based purely on the current modal energy level. This switching can cause sudden change in the control force(control gain), leading to deterioration of the vibration control response, and in a entreme cases even driving the system to instability. In the next section, we propose a new method that addresses the problems just mentioned.

3. A New Method in the Independent Modal Space Control

A modified IMSC is presented in this section. Developing an improved switching method that ensures the stability of the closed-loop system constitutes a key component of the proposed method. In contrast to the method of Baz and Poh in which the switching time is fixed for all modes, the switching time in the present method is dynamically determined by applying a new procedure that will be explained below. As before, the system modes are sorted based on the modal energy level. In the beginning, modes with large initial energy levels are selected for control. For each mode selected for control, an actuator is applied for the duration of the time constant (inverse of the real part of the eigenvalue) of that particular mode. The mode with the shortest time constant is up first for reappraisal. At that instant, the energy levels for all modes that are not currently being controlled (including the mode to which the actuator has just been taken off) are compared, and the mode possessing the largest energy level is selected for control. There is an additional requirement, however, when the actuator is being switched from one mode to another, as an abrupt change in the values of the control gain may cause discontinuity in the control force. Since this discontinuity will in general have an adverse effect on the system response, suitable time delays are introduced to avoid this possibility. The switching of the displacement feedback is designed to occur at the moment the modal displacement of the new mode becomes zero, while the switching of the velocity feedback occurs just as the modal velocity becomes zero. Introducing such time delays enhances the stability performance of the control system. When the next actuator is made available after the appropriate time has elapsed, the above procedure is repeated. In this way, a series of successive mode switchings occur as actuators are freed from one mode and applied to another.

Another key component is to modify IMSC as given below so that the control gain matrices G_p and G_V become symmetric, thus ensuring the stability of the closed-loop system:

$$f_C(t) = -K_p\nu_C(t) - K_V\dot{\nu}_C(t) - K_pB_{CC}^{-T}B_{UC}^T\nu_U(t) - K_VB_{CC}^{-T}B_{UC}^T\dot{\nu}_U(t). \quad (13)$$

The above equation combines the advantages of IMSC and the coupled control methodology. Applying Eq. (13) into Eq. (9), we obtain

$$f_U(t) = -B_{UC}B_{CC}^{-1}K_p\nu_C(t) - B_{UC}B_{CC}^{-1}K_V\dot{\nu}_C(t) - B_{UC}B_{CC}^{-1}K_pB_{CC}^{-T}B_{UC}^T\nu_U(t) - B_{UC}B_{CC}^{-1}K_VB_{CC}^{-T}B_{UC}^T\dot{\nu}_U(t). \quad (14)$$

A new modal control vector obtained from Eqs. (13) and (14) can be written as

$$f^*(t) = -\begin{bmatrix} K_p & K_pB_{CC}^{-T}B_{UC}^T \\ B_{UC}B_{CC}^{-1}K_p & B_{UC}B_{CC}^{-1}K_pB_{CC}^{-T}B_{UC}^T \end{bmatrix}\nu^*(t) - \begin{bmatrix} K_V & K_VB_{CC}^{-T}B_{UC}^T \\ B_{UC}B_{CC}^{-1}K_V & B_{UC}B_{CC}^{-1}K_VB_{CC}^{-T}B_{UC}^T \end{bmatrix}\dot{\nu}^*(t) = -G_p^*\nu^*(t) - G_V^*\dot{\nu}^*(t). \quad (15)$$

The new control gains G_p^* and G_V^* can readily be verified to be positive definite matrices.

To apply this new control scheme to the system, Eqs. (15) and (4) can be substituted into Eq. (1) as

$$I\ddot{\nu}(t) + \Lambda\nu(t) = f(t) = -\Gamma G_p^*\nu^*(t) - \Gamma G_V^*\dot{\nu}^*(t) = -\Gamma G_p^*\Gamma^T\nu(t) - \Gamma G_V^*\Gamma^T\dot{\nu}(t). \quad (16)$$

Hence, the closed-loop equation of motion of the vibration system can be expressed by

$$I\ddot{\nu}(t) + \Gamma G_V^*\Gamma^T\dot{\nu}(t) + (\Lambda + \Gamma G_p^*\Gamma^T)\nu(t) = 0, \quad (17)$$

where $\Gamma G_V^*\Gamma^T$ and $\Lambda + \Gamma G_p^*\Gamma^T$ are symmetric positive definite matrices.

4. Stability Analysis of the Closed-Loop System

The stability of the closed-loop system is now considered. Let us define the following candidate Lyapunov function for determining the asymptotic stability of the system:

$$V(t) = \frac{1}{2}\dot{\nu}^T(t)I\dot{\nu}(t) + \frac{1}{2}\nu^T(t)[\Lambda + \Gamma G_p^*\Gamma^T]\nu(t). \quad (18)$$

In the above equation, the first term denotes the kinetic energy, while the second term refers to the potential energy. Since I and $\Lambda + \Gamma G_p^*\Gamma^T$ are symmetric positive definite matrices, $V(t)$ is also positive definite. Differentiating Eq. (18) with respect to time, we obtain

$$\dot{V}(t) = \dot{\nu}^T(t)I\ddot{\nu}(t) + \nu^T(t)[\Lambda + \Gamma G_p^*\Gamma^T]\dot{\nu}(t). \quad (19)$$

Substituting Eq. (17) into (19) and rearranging, we obtain

$$\dot{V}(t) = -\dot{\nu}^T(t)\Gamma G_V^*\Gamma^T\dot{\nu}(t). \quad (20)$$

Except for the displacement and velocity feedback switching times, denoted t_s, the differentiation of $V(t)$ presents no problem. At the switching times, G_p and G_v need to be differentiated as at other times, but the resulting values can blow up to infinity. To forestall this possibility, the t_s are selected such that the displacement feedback is switched when $\nu(t_s) = 0$ and the velocity feedback switching occurs when $\dot{\nu}(t_s) = 0$. In this way, no discontinuity is present over the entire time domain. Since $\Gamma G_V^*\Gamma^T$ in Eq. (20) is positive definite, the condition that $\dot{V}(t_s) \leq 0$ is always satisfied, i. e., the right-hand side of Eq. (20) becomes negative semi-definite. Since $V(t)$ is positive definite while $\dot{V}(t)$ is negative semi-definite over the entire statedomain, the candidate becomes the Lyapunov function and the system is stable in the sense of Lyapunov. Furthermore, since $\dot{V}(t) = 0$ if and only if $\nu(t) = \dot{\nu}(t) = 0$, the system is asymptotically stable by the Invariant Set Theorem of La Salle (1991). To summarize, the above derivation shows that the proposed method guarantees asymptotic stability of the vibration control system even in the presence of mode switching.

5. An Example

To illustrate the method developed in the previous sections, a cantilevered beam will be used as an example. According to the Bernoulli-Euler model, the governing equation is

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right]+M(x)\frac{\partial^2 w(x,t)}{\partial t^2}=f(x,t).$$

It is assumed that the mass per unit length $M=1$, flexural rigidity $EI=10$, and length $l=10$. The number of actuators $m=2$, and the number of system modes $n=6$. The actuators are placed at $0.55l$ and $1.0l$ in order to avoid the nodal points of all 6 modes. The switching times t_s correspond to the moment at which the modal displacement and modal velocity become zero, counting from the moment the time constant has elapsed from the previous switching time. Applying a unit impulse at $x=9.8$, the simulated responses are presented in Figs. 1-4. For selected modes, the comparison of the modal displacements between the proposed

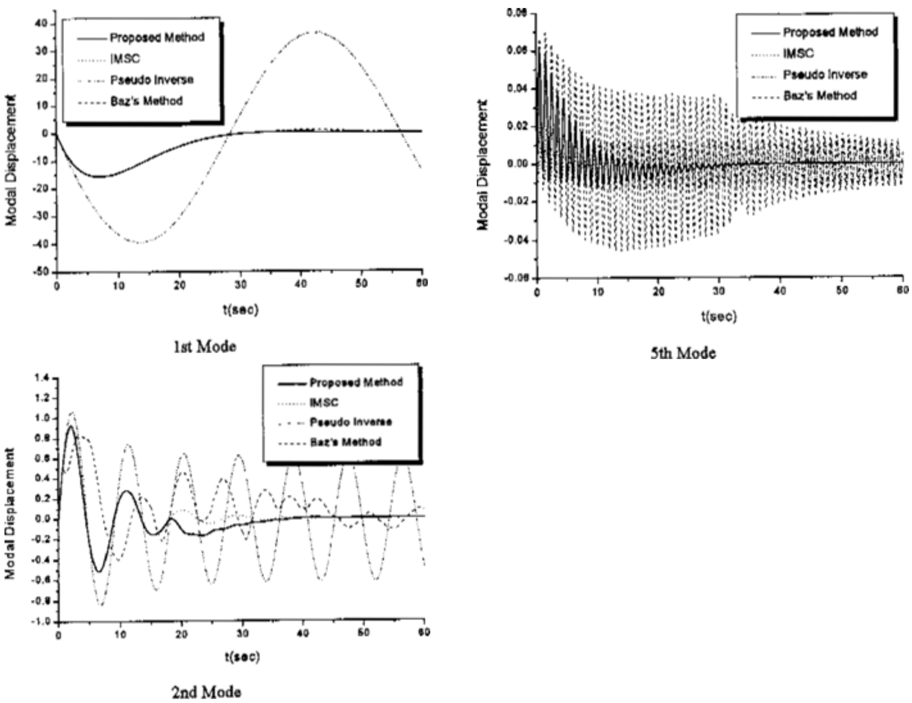

Fig. 1 Comparison of modal displacements.

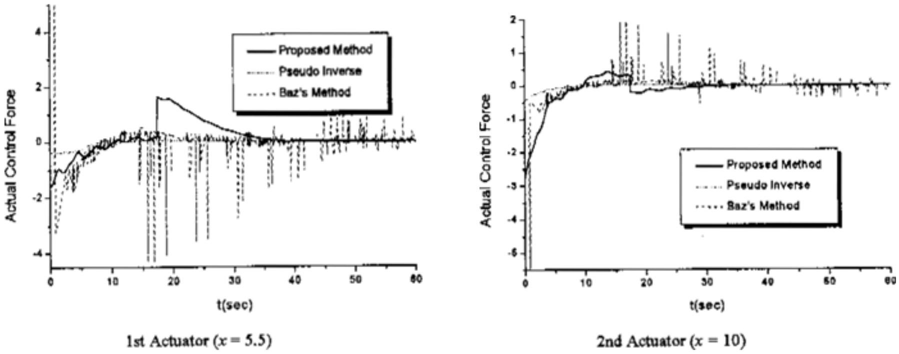

Fig. 2 Comparison of actual control forces.

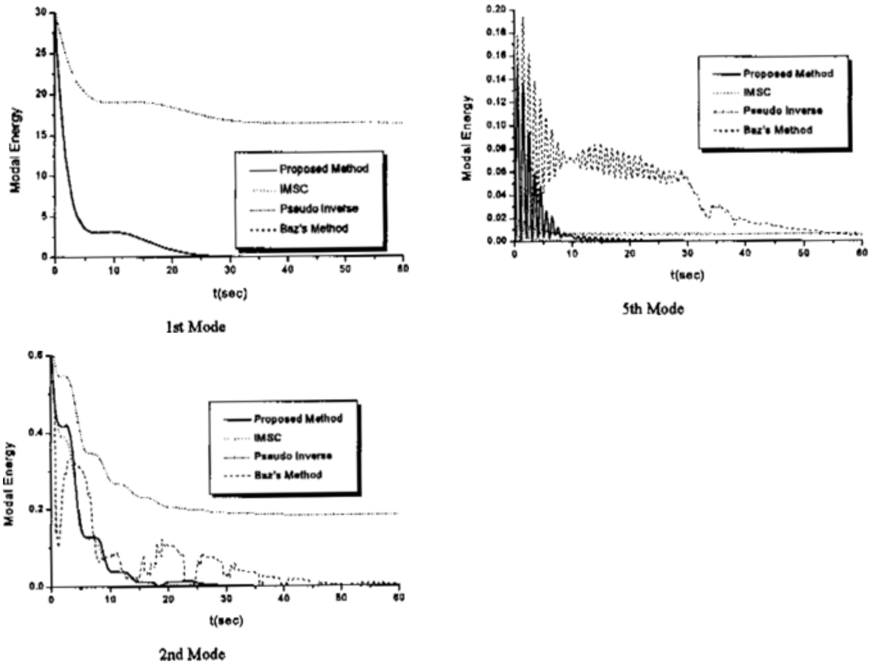

Fig. 3 Comparison of modal energy.

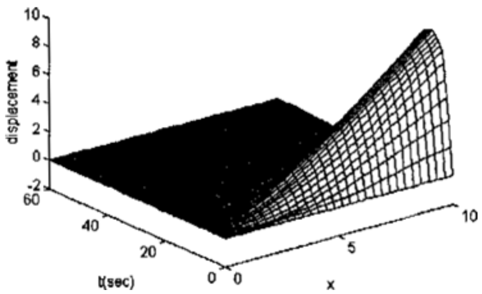

Fig. 4 Actual displacement response.

method, the pseudo-inverse method, the method of Baz and Poh, and the original IMSC ($m=n$) method is given in Fig. 1. The proposed method shows much improved vibration control response compared to the pseudo-inverse method and the method of Baz and Poh. Although the modal responses for only three modes are shown, similar results have been obtained for all other modes as well. Figure 2 compares the control forces, while Fig. 3 compares the modal energy levels for the four methods mentioned above. In Fig. 2, the method of Baz and Poh shows large discontinuities in the control force, resulting in serious degradation of the modal control responses. On the other hand, the magnitude of the control force computed is too small in the case of the pseudo-inverse method, which again reduces the modal control effectiveness. In the case of the proposed method, however, there is almost no discontinuity in the control force computed, and much improved vibration control is observed. In Fig. 3, the method of Baz and Poh shows large fluctuations in the modal energy level due to the discontinuities of the control force shown in Fig. 2. The figure also shows that the pseudo-inverse method is unsatisfactory with regard to vibration control. Although the performance of the proposed method is somewhat inferior to the original IMSC ($m=n$), it is clearly superior to the other two methods. Figure 4 shows the actual beam displacement as a function of time. It is seen that the vibration control response is quite satisfactory even with only two actuators.

6. Conclusions

The present investigation addresses the issue of reducing the number of actuators in independent modal space control. While using fewer actuators than the number of system modes, the proposed method can still be an effective tool for vibration control. The stability of the closed-loop system incorporating the switching algorithm is also demonstrated. Through an example, the proposed method is compared with previously published methods that also seek to reduce the number of actuators in independent modal space control. The results of the present work can be summarized as follows:

(1) An effective and stable switching algorithm that does not seriously degrade the performance of the vibration control system is developed.

(2) A procedure for determining the control force that ensures the stability of the vibration control system is devised. This method combines the "best" features of IMSC and coupled control techniques.

(3) A stability analysis of the closed-loop system incorporating the switching algorithm shows that the proposed method is asymptotically stable.

(4) Although the proposed method is slightly inferior to the original IMSC ($m=n$), the number of actuators can be significantly reduced. In terms of performance and stability, the present method is clearly superior to previously published methods that also seek a reduction in the number of actuators.

Although the effect of structural damping is not explicitly considered, the results of the present work can be applied to vibration control of damped structures as well.

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